4.4 Exercises: Total time in the system

Assume $T_1 \leq T_2 \leq T_3 \leq \ldots \leq T_n$

Greedy choice: Find an optimal schedule such that $J_1$ is scheduled first.

For any optimal schedule $S:\ [J_{i_1}, J_{i_2}, \ldots, J_{i_k}, \ldots, J_{i_n}]$  \[ 0 \quad f_{i_1}, f_{i_2}, f_{i_k}, f_{i_n} \]

Total time $\sum_{j=1}^{n} f_{i_j}$

If $J_{i_k} = J_1$ \Rightarrow done!

If $J_{i_k} \neq J_1$ \Rightarrow assume $J_{i_k} = J_1$

Swap $J_{i_k}$, $J_{i_{k-1}}$

$S'\ [J_{i_1}, J_{i_2}, \ldots, J_{i_{k-1}}, J_{i_k}, \ldots, J_{i_n}]$  \[ 0 \quad f_{i_1}, f_{i_2}, f_{i_{k-1}}, f_{i_k}, f_{i_n} \]

Compare total time of $S$ and $S'$ \Rightarrow

Compare: $f_{i_{k-1}}$ and $f^*$ (all other finishing times remain the same)

$f^* \leq f_{i_{k-1}}$ \Rightarrow Total time ($S'$) $\leq$ Total time ($S$) because $f_{i_k} = f_1$, which is smallest service time.

So, we can continue to swap $J_{i_k}$ to 1st position each time, the total time of new schedule is not more than previous schedule.
4.4 Exercises: Total time in the system

Optimal Substructure:

Problem $P$:

Assume $T_1 \leq T_2 \leq \ldots \leq T_n$

Available at time $t=0$

\[ \Downarrow \text{after considering} \]

\[ \Downarrow \]

Problem $P'$:

Assume $T_2 \leq \ldots \leq T_n$

Available at time $f_1 = T_1$

Claim: $S'$ is an optimal schedule for $P'$

If this is not true, let $S^*$ be an optimal schedule $S^*$

\[ \begin{bmatrix} J_{k_2} & J_{k_3} & \ldots & J_{k_n} \end{bmatrix} \]

\[ f_{k_1} & f_{k_2} & f_{k_3} & \ldots & f_{k_n} \]

\[ \sum_{j=2}^{n} f_{k_j} < \sum_{j=2}^{n} f_{i_j} \]

Total time($S^*$) < Total time($S'$)

Now, place $J_1$ in front of schedules $S^*$ and $S'$

\[ f_1 + \sum_{j=2}^{n} f_{k_j} < f_1 + \sum_{j=2}^{n} f_{i_j} \]

$\Rightarrow S$ is not optimal (contradiction)