HW Page 4 Question #3

Refer to counting sort: Step 1 to Step 3 \(O(n+k)\)

Query: \(c[i] - c[i-1] \quad O(1)\)

HW Page 5 Question #1

Assume \(\frac{w_1}{p_1} \geq \frac{w_2}{p_2} \geq \ldots \geq \frac{w_n}{p_n}\)

for any optimal schedule:

\[
S: J_1 \quad J_2 \quad \ldots \quad J_{k-1} \quad J_k \quad J_{k+1} \quad \ldots \quad J_n
\]

Total Cost: \((w_1 \times c_1) + (w_2 \times c_2) + \ldots + (w_n \times c_n)\)

Greedy Choice Property: \exists an optimal schedule such that \(J_i\) (longest ratio) is scheduled first.

If \(J_i = J_1\) \(\Rightarrow\) done

If \(J_i \neq J_1\) \(\Rightarrow\) assume \(J_{i+k1} = J_1\)

\[\text{Let swap } J_{ik} \text{ and } J_{i+k1} \text{ in } S\]

\[
S\quad \ldots \quad J_{ik} \quad J_{i+k1} \quad \ldots
\]

\[
C_{ik1} \quad C_k \quad C_{k1}
\]

\[
S^*\quad \ldots \quad J_{i+k1} \quad J_{ik} \quad \ldots
\]

\[
C_{k1} \quad C^* \quad C_{k+1}
\]

all jobs are in the same position except \(J_{ik}\) and \(J_{i+k1}\)

\[
C_k = C_{k1} + p_{ik}
\]

\[
C^* = C_{k1} + p_{ik1}
\]

\[
C_{k+1} = C_{k1} + p_i + p_{ik1}
\]

\[
\frac{w_{ik1}}{p_{ik1}} \text{ is the largest}
\]

Note: \(J_{i+k1} = J_i\) \(\Rightarrow\)

\[
\ln S \quad (C_k \times w_{ik}) + (C_{k1} \times w_{ik1})
\]

\[
\Rightarrow (C_{k1} + p_{ik}) w_{ik} + (C_{k1} + p_i + p_{ik1}) w_{ik1}
\]

\[
\ln S^* \quad (C^* \times w_{ik1}) + (C_{k1} \times w_i)
\]

\[
\Rightarrow (C_{k1} + p_{ik1}) w_{ik1} + (C_{k1} + p_i + p_{ik1}) w_i
\]

\[\text{If } \ln S \geq 0 \quad \text{by } 8\]

\[\text{If } \ln S^* \geq 0 \quad \text{by } 8\]

So, we can continue to swap \(J_{i+k1}\) to the 1st position.
The final schedule, total cost \leq \text{total cost of } S \\
so it is an optimal schedule

Rabin-Karp Alg only work for 0's and 1's
Let \( P = 0010010 \), \( t = 1001100100100 \)
\( |P| = 7 \) use \( h(i) \) formula as given in class note
\[
h(P) = (0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0) \mod 7
\]
\[= 18 \mod 7 = 4 \]
Compute Fingerprint table: for \( h(0) \) to \( h(6) \)
\[
h(0) = (2^6 + 2^3 + 2^2) \mod 7 = 76 \mod 7 = 6
\]
\[
h(1) = [1 + 2(6 - 2^6)] \mod 7 = -115 \mod 7 = 4
\]
\[
h(2) = [0 + 2(4 - 0)] \mod 7 = 8 \mod 7 = 1
\]
\[
h(3) = [0 + 2(1 - 0)] \mod 7 = 2 \mod 7 = 2
\]
\[
h(4) = [1 + 2(2 - 2^6)] \mod 7 = -123 \mod 7 = 3
\]
\[
h(5) = [0 + 2(3 - 2^6)] \mod 7 = -122 \mod 7 = 4
\]
\[
h(6) = [0 + 2(4 - 0)] \mod 7 = 8 \mod 7 = 1
\]
\[
h(p) = h(5), \text{ so } P = t[5...11]